Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

9999675145

MATHEMATICS **9709/21**

Paper 2 Pure Mathematics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

[Turn over

1 It is given that θ is an acute angle in degrees such that $\sin\theta = \frac{2}{3}$.

Find the exact value of $\sin(\theta + 60°)$. [3]

2 A curve has equation $y = 3\tan\frac{1}{2}x\cos 2x$.

Find the gradient of the curve at the point for which $x = \frac{1}{3}\pi$. [5]

3 **(a)** Find $\int_4^{10} \frac{4}{2x-5}\,\mathrm{d}x$, giving your answer in the form $\ln a$, where a is an integer. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the exact value of $\int_4^{10} \mathrm{e}^{2x-5}\,\mathrm{d}x$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

4 **(a)** Sketch, on the same diagram, the graphs of $y = |3x - 5|$ and $y = 2x + 7$. [2]

(b) Solve the equation $|3x - 5| = 2x + 7$. [3]

.....

.....

.....

.....

.....

.....

(c) Hence solve the equation $\left|3^{y+1} - 5\right| = 2 \times 3^{y} + 7$, giving your answer correct to 3 significant figures. [2]

.....

.....

.....

.....

.....

.....

5 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = 6x^3 + ax^2 + bx - 20,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$ and that the remainder is -11 when $\mathrm{p}(x)$ is divided by $(x + 1)$.

(a) Find the values of a and b. [5]

(b) Hence factorise $p(x)$, and determine the exact roots of the equation $p(3x) = 0$. [4]

6 **(a)** Show that $\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) \equiv 4 + 6\cos\theta - 4\cos^2\theta$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Solve the equation

$$\operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta) + 3 = 0$$

for $-\pi < \theta < 0$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find $\int \operatorname{cosec}\theta(3\sin 2\theta + 4\sin^3\theta)\,\mathrm{d}\theta$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 The curve with equation $e^{2x} - 18x + y^3 + y = 11$ has a stationary point at (p, q).

(a) Find the exact value of p. [4]

(b) Show that $q = \sqrt[3]{2 + 18 \ln 3 - q}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

(c) Show by calculation that the value of q lies between 2.5 and 3.0. [2]

.....

.....

.....

.....

.....

(d) Use an iterative formula, based on the equation in **(b)**, to find the value of q correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....